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Vibration of Orthotropic Rectangular Plates Under the Action of Moving Distributed Masses and Resting on a Variable Elastic Pasternak Foundation with Clamped End Conditions

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Abstract*— *This work investigates the vibration of orthotropic rectangular plate resting on a variable elastic Pasternak foundation under the action of moving distributed masses. The governing equation is a fourth order partial differential equation with variable and singular co-efficients. The solutions to the problem are obtained by transforming the fourth order partial differential equation for the problem to a set of coupled second order ordinary differential equations using the technique of Shadnam et al[11] which are then simplified using modified asymptotic method of Struble. The closed form solution is analyzed, resonance conditions are obtained and the results are presented in plotted curves for both cases of moving distributed mass and moving distributed force.

I. INTRODUCTION

The problems related to thin structural bodies (rods, beams, plates,and shells) with other bodies have widespread application in various fields of science and technology. The physical phenomena involved in the impact event include structural responses, contact effects and wave propagation. The problems associated with these are always topical issues in the field of applied mechanics. Since these problems belong to the problems related to dynamic contact interaction, their solution is connected with cumbersome mathematical tasks. To this end , several researchers had worked and some are still working on the dynamic behavior of orthotropic rectangular plates. Ambartsumian [1] examined the five fundamental differential equations describing the equilibrium of an orthotropic plate with a cylindrical anisotropy for the case when all radial planes crossing the axis of anisotropy are the planes of elastic symmetry. Sveklo [2] suggested the contact theory for two anisotropic bodies under compression according to which the contact pressure is distributed over an elliptical contact region. The same structural effects are also true of the concrete slab in a composite girder bridge, but the steel orthotropic deck is considerably lighter, and therefore allows longer span bridges to be more efficiently designed. Awodola [3] studied the effect of plate parameters on the vibrations under moving masses of elastically supported plate resting on bi-parametric foundation with stiffness variation.Szekrenyes [4] investigated the interface fracture in orthotropic composite plates using second order shear deformation theory. Yan [5] proposed elastic orthotropic models and used these in the nonlinear analysis of concrete structures subjected to monotonic or pseudo dynamic

loading. Since these models can appropriately describe the strain softening behavior of concrete beyond the peak stress and show good agreement with the strength envelope obtained from experimental results Hu and Yao [6] studied the vibration solutions of rectangular orthotropic plates by symplectic geometry method. In the same vien, Alshaya, Hunt and Rowlands [7] examined stresses and strains in thick perforated orthotropic plates. Gbadeyan and Dada [8] found the natural frequency of rectangular plates traversed by moving concentrated masses. Awodola and Adeoye [9] investigated the behavior of simply supported orthotropic rectangular plate by applying the technique of variable separable. Adeoye and Awodola [10] studied the dynamic behavior of orthotropic rectangular plate with clamped-clamped boundary conditions by making use of the technique of Shadnam Due to inability of researchers to solve orthotropic plates problems by analytical methods, this work aims at solving the governing equation by analytical solution and also considers the effect of the flexural rigidities in both x and y directions.

II. GOVERNING EQUATION

The dynamic transverse displacement $W(x,y,t)$ of orthotropic rectangular plates when it is resting on a bi-parametric elastic foundation and traversed by distributed mass M_r moving with constant velocity c_r along a straight line parallel to the x-axis issuing from point y = s on the y-axis with flexural rigidities D_x and D_y is governed by the fourth order partial differential equation given as

$$D_x \frac{\partial^4}{\partial x^4} W(x,y,t) + 2B \frac{\partial^4}{\partial x^2 \partial y^2} W(x,y,t) + D_y \frac{\partial^4}{\partial y^4} W(x,y,t) + \mu \frac{\partial^2}{\partial t^2} W(x,y,t) - \rho h R_0 [\frac{\partial^4}{\partial x^2 \partial t^2} W(x,y,t) + \frac{\partial^4}{\partial y^2 \partial t^2} W(x,y,t)] + K_0(4x - 3x^2 + x^3) W(x,y,t) + S_0(-13 + 12x + 3x^2) \frac{\partial}{\partial x} W(x,y,t) - S_0(12 - 13x + 6x^2 + x^3)[\frac{\partial^2}{\partial x^2} W(x,y,t) + \frac{\partial^2}{\partial y^2} W(x,y,t)] - \sum_{r=1}^{N} [M_r g H(x - ct) H(y - s) - M_r (\frac{\partial^2}{\partial t^2} W(x,y,t) + 2c_r \frac{\partial^2}{\partial x \partial t} W(x,y,t) + c_r^2 \frac{\partial^2}{\partial x^2} W(x,y,t)) H(x - c_r t) H(y - s)] = 0 \quad (2.1)$$

where D_x and D_y are the flexural rigidities of the plate along x and y axes respectively.

$$D_x = \frac{E_x h^3}{12(1 - \nu_x \nu_y)}, \quad D_y = \frac{E_y h^3}{12(1 - \nu_x \nu_y)}, \quad B = D_x D_y + \frac{G_o h^3}{6} \quad (2.2)$$

E_x and E_y are the Young's moduli along x and y axes respectively, G_o is the rigidity modulus, ν_x and ν_y are Poisson's ratios for the material such that $E_x \nu_y = E_y \nu_x$, ρ is the mass density per unit volume of the plate, h is the plate thickness, t is the time, x and y are the spatial coordinates in x and y directions respectively, R_o is the rotatory inertia correction factor, K_o is the foundation constant, S_o shear modulus and g is the acceleration due to gravity, H(.) is the Heaviside function.

Rewriting equation (2.1), one obtains

$$\mu \frac{\partial^2}{\partial t^2} W(x,y,t) + \mu \omega_n^2 W(x,y,t) = \rho h R_0 [\frac{\partial^4}{\partial x^2 \partial t^2} W(x,y,t) + \frac{\partial^4}{\partial y^2 \partial t^2} W(x,y,t)] - 2B \frac{\partial^4}{\partial x^2 \partial y^2} W(x,y,t) - D_y \frac{\partial^4}{\partial y^4} W(x,y,t) - D_x \frac{\partial^4}{\partial x^4} W(x,y,t) + \mu \omega_n^2 W(x,y,t) - K_0(4x - 3x^2 + x^3) W(x,y,t) + G_o(-13 + 12x + 3x^2) \frac{\partial}{\partial x} W(x,y,t) - G_0(12 - 13x + 6x^2 + x^3)[\frac{\partial^2}{\partial x^2} W(x,y,t) + \frac{\partial^2}{\partial y^2} W(x,y,t)] + \sum_{r=1}^{N} [M_r g H(x - c_r t) H(y - s) - M_r (\frac{\partial^2}{\partial t^2} W(x,y,t) + 2c_r \frac{\partial^2}{\partial x \partial t} W(x,y,t) + c_r^2 \frac{\partial^2}{\partial x^2} W(x,y,t)) H(x - c_r t) H(y - s)] \quad (2.3)$$

Simplifying equation (2.3) further, one obtains

$$\frac{\partial^2}{\partial t^2} W(x,y,t) + \omega_n^2 W(x,y,t) = \sum_{r=1}^{N} [R_0 (\frac{\partial^4}{\partial x^2 \partial t^2} W(x,y,t) + \frac{\partial^4}{\partial y^2 \partial t^2} W(x,y,t)) - \frac{2B}{\mu} \frac{\partial^4}{\partial x^2 \partial y^2} W(x,y,t) - \frac{D_y}{\mu} \frac{\partial^4}{\partial y^4} W(x,y,t) - \frac{D_x}{\mu} \frac{\partial^4}{\partial x^4} W(x,y,t) + \omega_n^2 W(x,y,t) - \frac{K_0}{\mu}(4x - 3x^2 + x^3) W(x,y,t) + \frac{G_o}{\mu}(-13 + 12x + 3x^2) \frac{\partial}{\partial x} W(x,y,t) - \frac{G_o}{\mu}(12 - 13x + 6x^2 + x^3)(\frac{\partial^2}{\partial x^2} W(x,y,t) + \frac{\partial^2}{\partial y^2} W(x,y,t)) + \sum_{r=1}^{N} (\frac{M_r}{\mu} g H(x - c_r t) H(y - s) - \frac{M_r}{\mu} (\frac{\partial^2}{\partial t^2} W(x,y,t) + 2c_r \frac{\partial^2}{\partial x \partial t} W(x,y,t) + c_r^2 \frac{\partial^2}{\partial x^2} W(x,y,t)) H(x - c_r t) H(y - s))] \quad (2.4)$$

where ω_n^2 is the natural frequencies, $n = 1,2,3,\ldots$

The initial conditions, without any loss of generality, is taken as

$$W(x,y,t) = 0 = \frac{\partial}{\partial t}W(x,y,t) \tag{2.5}$$

III. ANALYTICAL APPROXIMATE SOLUTION

In order to solve equation (2.4), one applies technique of Shadnam et al which requires that the deflection of the plates be in series form as

$$W(x,y,t) = \sum_{n=1}^{N} \Psi_n(x,y)Q_n(t) \tag{3.1}$$

where

$$\Psi_n(x,y) = \Psi_{jm}(x)\Psi_{hm}(y)$$

$$\Psi_{jm}x = \sin\zeta_{jm}x + A_{jm}\cos\zeta_{jm}x + B_{jm}\sinh\zeta_{jm}x + C_{jm}\cosh\zeta_{jm}x$$

$$\Psi_{hm}y = \sin\varphi_{hm}y + A_{hm}\cos\varphi_{hm}y + B_{hm}\sinh\varphi_{hm}y + C_{hm}\cosh\varphi_{hm}y$$

$$\zeta_{jm} = \frac{\phi_{jm}}{L_x}, \quad \varphi_{hm} = \frac{\phi_{hm}}{L_y}$$

The right hand side of equation (2.4), taken into account equation (3.1), written in the form of series takes the form

$$\begin{aligned}&\sum_{n=1}^{\infty} [R_0(\frac{\partial^2}{\partial x^2}\Psi_n(x,y)\ddot{Q}_n(t) + \frac{\partial^4}{\partial y^2}\Psi_n(x,y)\ddot{Q}_n(t)) - \frac{2B}{\mu}\frac{\partial^4}{\partial x^2\partial y^2}\Psi_n(x,y)Q_n(t) - \frac{D_y}{\mu}\frac{\partial^4}{\partial y^4}\\ &\Psi_n(x,y)Q_n(t) - \frac{D_x}{\mu}\frac{\partial^4}{\partial x^4}\Psi_n(x,y)Q_n(t) + \omega_n^2\Psi_n(x,y)Q_n(t) - \frac{K_0}{\mu}(4x - 3x^2 + x^3)\Psi_n(x,y)\\ &Q_n(t) + \frac{G_o}{\mu}(-13 + 12x + 3x^2)\frac{\partial}{\partial x}\Psi_n(x,y)Q_n(t) - \frac{G_0}{\mu}(12 - 13x + 6x^2 + x^3)(\frac{\partial^2}{\partial x^2}\Psi_n(x,y)\\ &Q_n(t) + \frac{\partial^2}{\partial y^2}\Psi_n(x,y)Q_n(t)) + \sum_{r=1}^{N} (\frac{M_r}{\mu}gH(x - c_r t)H(y - s) - \frac{M_r}{\mu}(\Psi_n(x,y)\ddot{Q}_n(t) +\\ &2c\frac{\partial}{\partial x}\Psi_n(x,y)\dot{Q}_n(t) + c_r^2\frac{\partial^2}{\partial x^2}\Psi_n(x,y)Q_n(t))H(x - c_r t)H(y - s))] = \sum_{n=1}^{N} \Psi_n(x,y)\Theta_n(t)\end{aligned} \tag{3.2}$$

Multiplying both sides of equation (3.2) by $\Psi_m(x,y)$ and integrating on area A of the plate and considering the orthogonality of $\Psi_n(x,y)$, one obtains

$$\begin{aligned}\Theta_n(t) = &\frac{1}{\theta^*}\sum_{n=1}^{\infty} \int_A [R_0(\frac{\partial^2}{\partial x^2}\Psi_n(x,y)\ddot{Q}_n(t) + \frac{\partial^4}{\partial y^2}\Psi_n(x,y)\ddot{Q}_n(t)) - \frac{2B}{\mu}\frac{\partial^4}{\partial x^2\partial y^2}\Psi_n(x,y)\\ &Q_n(t) - \frac{D_y}{\mu}\frac{\partial^4}{\partial y^4}\Psi_n(x,y)Q_n(t) - \frac{D_x}{\mu}\frac{\partial^4}{\partial x^4}\Psi_n(x,y)Q_n(t) + \omega_n^2\Psi_n(x,y)Q_n(t) - \frac{K_0}{\mu}(4x -\\ &3x^2 + x^3)\Psi_n(x,y)Q_n(t) + \frac{G_o}{\mu}(-13 + 12x + 3x^2)\frac{\partial}{\partial x}\Psi_n(x,y)Q_n(t) - \frac{G_0}{\mu}(12 - 13x\\ &+6x^2 + x^3)(\frac{\partial^2}{\partial x^2}\Psi_n(x,y)Q_n(t) + \frac{\partial^2}{\partial y^2}\Psi_n(x,y)Q_n(t)) + \sum_{r=1}^{N} (\frac{M_r}{\mu}gH(x - c_r t)H(y - s)\\ &-\frac{M_r}{\mu}(\Psi_n(x,y)\ddot{Q}_n(t) + 2c\frac{\partial}{\partial x}\Psi_n(x,y)\dot{Q}_n(t) + c_r^2\frac{\partial^2}{\partial x^2}\Psi_n(x,y)Q_n(t))H(x - c_r t)H(y - s)\\ &)]\Psi_m(x,y)dA\end{aligned} \tag{3.3}$$

and zero when $n \neq m$

where

$$\theta^* = \int_A \Psi_n^2(x,y)dA \tag{3.4}$$

Making use of equation (3.3) and taking into account equation (3.2), equation (2.4) can be written as

$$\Psi_n(x,y)[\omega_n^2 Q_n(t)+\ddot{Q}_n(t)]=\frac{\Psi_n(x,y)}{\theta^*}\sum_{q=1}^{\infty}\int_A [R_0(\frac{\partial^2\Psi_q(x,y)}{\partial x^2}\Psi_m(x,y)\ddot{Q}_q(t)+\frac{\partial^2\Psi_q(x,y)}{\partial y^2} \Psi_m(x,y)\ddot{Q}_q(t))-\frac{2B}{\mu}\frac{\partial^2\Psi_q(x,y)}{\partial x^2\partial y^2}\Psi_m(x,y)Q_q(t)-\frac{D_y}{\mu}\frac{\partial^4\Psi_q(x,y)}{\partial y^4}\Psi_m(x,y)Q_q(t)+\frac{D_x}{\mu}\frac{\partial^4\Psi_q(x,y)}{\partial x^4} \Psi_m(x,y)Q_q(t)-\frac{K_0}{\mu}(4x-3x^2+x^3)\Psi_q(x,y)\Psi_m(x,y)Q_q(t)+\frac{G_0}{\mu}(-13+12x+3x^2)\frac{\partial\Psi_q(x,y)}{\partial x} \Psi_m(x,y)Q_q(t)-\frac{G_0}{\mu}(12-13x+6x^2+x^3)(\frac{\partial^2\Psi_q(x,y)}{\partial x^2}\Psi_m(x,y)Q_q(t)+\frac{\partial^2\Psi_q(x,y)}{\partial y^2}\Psi_m(x,y) Q_q(t))+\sum_{r=1}^{N}(\frac{M_r}{\mu}g\Psi_m(x,y)H(x-c_rt)H(y-s)-\frac{M_r}{\mu}(\Psi_q(x,y)\Psi_m(x,y)\ddot{Q}_q(t)+2c_r \frac{\partial\Phi_q(x,y)}{\partial x}\Phi_m(x,y)\dot{Q}_q(t)+c_r^2\frac{\partial^2\Phi_q(x,y)}{\partial x^2}\Phi_m(x,y)Q_q(t))H(x-c_rt)H(y-s))]dA \tag{3.5}$$

On further simplification of equation (3.5), one obtains

$$\ddot{Q}_n(t)+\omega_n^2 Q_n(t)=\frac{1}{\theta^*}\sum_{q=1}^{\infty}\int_A [R_0(\frac{\partial^2\Psi_q(x,y)}{\partial x^2}\Psi_m(x,y)\ddot{Q}_q(t)+\frac{\partial^2\Psi_q(x,y)}{\partial y^2}\Psi_m(x,y)\ddot{Q}_q(t)) -\frac{2B}{\mu}\frac{\partial^2\Psi_q(x,y)}{\partial x^2\partial y^2}\Psi_m(x,y)Q_q(t)-\frac{D_y}{\mu}\frac{\partial^4\Psi_q(x,y)}{\partial y^4}\Psi_m(x,y)Q_q(t)-\frac{D_x}{\mu}\frac{\partial^4\Psi_q(x,y)}{\partial x^4}\Psi_m(x,y) Q_q(t)+\omega_q^2\Psi_q(x,y)\Psi_m(x,y)Q_n(t)-\frac{K_0}{\mu}(4x-3x^2+x^3)\Psi_q(x,y)\Psi_m(x,y)Q_q(t)+\frac{G_0}{\mu}(-13 +12x+3x^2)\frac{\partial\Psi_q(x,y)}{\partial x}\Psi_m(x,y)Q_q(t)-\frac{G_0}{\mu}(12-13x+6x^2+x^3)(\frac{\partial^2\Psi_q(x,y)}{\partial x^2}\Psi_m(x,y)Q_q(t) +\frac{\partial^2\Psi_q(x,y)}{\partial y^2}\Psi_m(x,y)Q_q(t))+\sum_{r=1}^{N}(\frac{M_r}{\mu}g\Psi_m(x,y)H(x-c_rt)H(y-s)-\frac{M_r}{\mu}(\Psi_q(x,y)\Psi_m(x,y) \ddot{Q}_q(t)+2c_r\frac{\partial\Phi_q(x,y)}{\partial x}\Phi_m(x,y)\dot{Q}_q(t)+c_r^2\frac{\partial^2\Phi_q(x,y)}{\partial x^2}\Phi_m(x,y)Q_q(t))H(x-c_rt)H(y-s))]dA \tag{3.6}$$

The system of equations in equation (3.6) is a set of coupled ordinary differential equations where $H(x-c_rt)$ and $H(y-s)$ are the Heaviside functions which are defined as

$$H(x-c_rt)=\begin{cases}1, for\ x\geq c_rt\\0, for\ x<c_rt\end{cases},\qquad H(y-s)=\begin{cases}1, for\ y\geq s\\0, for\ y<s\end{cases} \tag{3.7}$$

With the properties

$$(i)\frac{d}{dx}[H(x-c_rt)]=\delta(x-c_rt),\qquad \frac{d}{dy}[H(y-s)]=\delta(y-s) \tag{3.8}$$

$$(ii)f(x)H(x-c_rt)=\begin{cases}f(x), for\ x\geq c_rt\\0, for\ x<c_rt\end{cases},\qquad f(y)H(y-s)=\begin{cases}f(y), for\ y\geq s\\0, for\ y<s\end{cases} \tag{3.9}$$

Using the Fourier series representation, the Heaviside functions take the form

$$H(x-c_rt)=\frac{1}{4}+\frac{1}{\pi}\sum_{r=1}^{N}\frac{\sin(2n+1)\pi(x-c_rt)}{2n+1}, 0<x<1 \tag{3.10}$$

$$H(y-s)=\frac{1}{4}+\frac{1}{\pi}\sum_{r=1}^{N}\frac{\sin(2n+1)\pi(y-s)}{2n+1}, 0<y<1 \tag{3.11}$$

On putting equations (3.7) to (3.11) into equation (3.6) and simplifying, one obtains

$$\ddot{Q}_n(t)+\omega_n^2Q_n(t)-\frac{1}{\theta^*}\sum_{q=1}^{\infty}[R_0T_1\ddot{Q}_q(t)-\frac{2B}{\mu}T_2Q_q(t)-\frac{D_y}{\mu}T_3Q_q(t)-\frac{D_x}{\mu}T_4Q_q(t)+(\omega_q^2F_4^*- \frac{K_0}{\mu}F_5^*)Q_q(t)+\frac{G_0}{\mu}(T_6+T_7)Q_q(t)-\sum_{r=1}^{N}\frac{M_r}{\mu}((T_8+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_1^*\frac{\cos(2j+1)\pi c_rt}{2j+1}-\sum_{j=1}^{\infty}E_2^* \frac{\sin(2j+1)\pi c_rt}{2j+1})(\sum_{k=1}^{\infty}E_3^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_4^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}E_5^*\frac{\cos(2j+1)\pi c_rt}{2j+1} -\sum_{j=1}^{\infty}E_6^*\frac{\sin(2j+1)\pi c_rt}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}E_7^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_8^*\frac{\sin(2k+1)\pi s}{2k+1}))\ddot{Q}_q(t)+ 2c_r(T_9+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_9^*\frac{\cos(2j+1)\pi c_rt}{2j+1}-\sum_{j=1}^{\infty}E_{10}^*\frac{\sin(2j+1)\pi c_rt}{2j+1})(\sum_{k=1}^{\infty}E_{11}^*\frac{\cos(2k+1)\pi s}{2k+1}$$

$$-\sum_{k=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{13}^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_{14}^* \frac{\sin(2j+1)\pi c_r t}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi s}{2k+1}))\dot{Q}_q(t) + c_r^2(T_{10} + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi c_r t}{2j+1})(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi c_r t}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi s}{2k+1}))Q_q(t))] = \sum_{q=1}^{\infty} \sum_{r=1}^{N} \frac{M_r g}{\mu\theta^*} \Psi_m(ct)\Psi_m(s) \quad (3.12)$$

which is the transformed equation governing the problem of an orthotropic rectangular plate resting on bi-parametric elastic foundation.

where

$$T_1 = \int_A [\frac{\partial^2}{\partial x^2}\Psi_q(x,y)\Psi_m(x,y) + \frac{\partial^2}{\partial y^2}\Psi_q(x,y)\Psi_m(x,y)]dA \quad (3.13)$$

$$T_2 = \int_A \frac{\partial^2}{\partial x^2}[\frac{\partial^2}{\partial x^2}\Psi_q(x,y)]\Psi_m(x,y)dA \quad (3.14)$$

$$T_3 = \int_A \frac{\partial^4}{\partial y^4}[\Psi_q(x,y)]\Psi_m(x,y)dA \quad (3.15)$$

$$T_4 = \int_A \frac{\partial^4}{\partial x^4}[\Psi_q(x,y)]\Psi_m(x,y)dA \quad (3.16)$$

$$F_4^* = \int_A \Psi_q(x,y)\Psi_m(x,y)dA \quad (3.17)$$

$$T_5 = 4U_1 - 3U_2 + U_3, \qquad T_6 = -13A_1 + 12A_2 + 3A_3 \quad (3.18)$$

$$T_7 = 12f_1 - 13f_2 + 6f_3 + f_4 + 12f_5 - 13f_6 + 6f_7 + f_8 \quad (3.19)$$

$$T_8 = \frac{1}{16}\int_A \Psi_q(x,y)\Psi_m(x,y)dA \quad (3.20)$$

$$E_1^* = \int_A \Psi_q(x,y)\Psi_m(x,y)\sin(2j+1)\pi x dA \quad (3.21)$$

$$E_2^* = \int_A \Psi_q(x,y)\Psi_m(x,y)\cos(2j+1)\pi x dA \quad (3.22)$$

$$E_3^* = \int_A \Psi_q(x,y)\Psi_m(x,y)\sin(2k+1)\pi y dA \quad (3.23)$$

$$E_4^* = \int_A \Psi_q(x,y)\Psi_m(x,y)\cos(2k+1)\pi y dA \quad (3.24)$$

$$E_5^* = E_1^*, \quad E_6^* = E_2^*, \quad E_7^* = E_3^*, \quad E_8^* = E_4^* \quad (3.25)$$

$$T_9 = \frac{1}{16}\int_A \frac{\partial}{\partial x}\Psi_q(x,y)\Psi_m(x,y)dA \quad (3.26)$$

$$E_9^* = \int_A \frac{\partial}{\partial x}(\Psi_q(x,y))\Psi_m(x,y)\sin(2j+1)\pi x dA \quad (3.27)$$

$$E_{10}^* = \int_A \frac{\partial}{\partial x}(\Psi_q(x,y))\Psi_m(x,y)\cos(2j+1)\pi x dA \quad (3.28)$$

$$E_{11}^* = \int_A \frac{\partial}{\partial x}(\Psi_q(x,y))\Psi_m(x,y)\sin(2k+1)\pi y dA \quad (3.29)$$

$$E_{12}^* = \int_A \frac{\partial}{\partial x}\Psi_q(x,y)\Psi_m(x,y)\cos(2k+1)\pi y dA \quad (3.30)$$

$$E_{13}^* = E_9^*, \quad E_{14}^* = E_{10}^*, \quad E_{15}^* = E_{11}^*, \quad E_{16}^* = E_{12}^* \quad (3.31)$$

$$T_{10} = \frac{1}{16}\int_A \frac{\partial^2}{\partial x^2}(\Psi_q(x,y))\Psi_m(x,y)dA \quad (3.32)$$

$$E_{17}^* = \int_A \frac{\partial^2}{\partial x^2}(\Psi_q(x,y))\Psi_m(x,y)\sin(2j+1)\pi x dA \quad (3.33)$$

$$E_{18}^* = \int_A \frac{\partial^2}{\partial x^2}(\Psi_q(x,y))\Psi_m(x,y)\cos(2j+1)\pi x dA \quad (3.34)$$

$$E_{19}^* = \int_A \Psi_q(x,y)\Psi_m(x,y)\sin(2k+1)\pi y dA \quad (3.35)$$

$$E_{20}^* = \int_A \frac{\partial^2}{\partial x^2}(\Psi_q(x,y))\Psi_m(x,y)\cos(2k+1)\pi y dA \quad (3.36)$$

$$E_{21}^* = E_{17}^*, \quad E_{22}^* = E_{18}^*, \quad E_{23}^* = E_{19}^*, \quad E_{24}^* = E_{20}^* \quad (3.37)$$

$\Psi_m(x,y)$ is assumed to be the products of functions $\Psi_{pm}(x)\Psi_{bm}(y)$ which are the beam functions in the directions of x and y axes respectively. That is

$$\Psi_m(x,y) = \Psi_{pm}(x)\Psi_{bm}(y) \quad (3.38)$$

where

$$\Phi_m(x) = \sin\frac{\Gamma_m x}{L_x} + A_m\cos\frac{\Gamma_m x}{L_x} + B_m\sinh\frac{\Gamma_m x}{L_x} + C_m\cosh\frac{\Gamma_m x}{L_x} \quad (3.39)$$

$$\Phi_m(y) = \sin\frac{\Gamma_m y}{L_y} + A_m\cos\frac{\Gamma_m y}{L_y} + B_m\sinh\frac{\Gamma_m y}{L_y} + C_m\cosh\frac{\Gamma_m y}{L_y} \quad (3.40)$$

where A_{pm}, B_{pm}, C_{pm}, A_{bm}, B_{bm} and C_{bm} are constants determined by the boundary conditions. And Ψ_{pm} and Ψ_{bm} are called the mode frequencies

where

$$\lambda_{pm} = \frac{\xi_{pm}}{L_x}, \qquad \lambda_{bm} = \frac{\xi_{bm}}{L_y} \quad (3.41)$$

Considering a unit mass, equation (3.12) can be re-written as

$$\ddot{Q}_n(t) + \omega_n^2 Q_n(t) - \frac{1}{\theta^*}\sum_{q=1}^{\infty} [R_0 T_1 \ddot{Q}_q(t) - \frac{2B}{\mu} T_2 Q_q(t) - \frac{D_y}{\mu} T_3 Q_q(t) - \frac{D_x}{\mu} T_4 Q_q(t) + (\omega_q^2 F_4^* - \frac{K_0}{\mu} T_5) Q_q(t) + \frac{G_0}{\mu}(T_6 + T_7) Q_q(t) - \alpha\rho((T_8 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_2^* \frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_3^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_5^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_6^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi s}{2k+1}))\ddot{Q}_q(t) + 2c(T_9 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_{11}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{13}^*$$

$$\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{14}^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi s}{2k+1}))\dot{Q}_q(t) + c^2(T_{10} + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} B_{20}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1}$$

$$- \sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi s}{2k+1}))Q_q(t))] = \sum_{q=1}^{\infty}\sum_{r=1}^{N} \frac{Mg}{\mu\theta^*}\Psi_m(ct)\Psi_m(s) \tag{3.42}$$

equation (3.42) is the fundamental equation of the problem. where

$$\alpha = \frac{M}{\mu\rho}, \quad \rho = L_x L_y \tag{3.43}$$

$$\Psi_m(ct) = \sin\chi_m t + A_m \cos\chi_m t + B_m \sinh\chi_m t + C_m \cosh\chi_m t \tag{3.44}$$

$$\Psi_m(s) = \sin\nu_m + A_m \cos\nu_m + B_m \sinh\nu_m + C_m \cosh\nu_m \tag{3.45}$$

$$\chi_m = \frac{\phi_m c}{L_x}, \quad \nu_m = \frac{\phi_m s}{L_y} \tag{3.46}$$

3.1 Orthotropic Rectangular Plate Traversed by a Moving Force

In moving force, we account for only the load being transferred to the structure. In this case, the inertia effect is negligible. Setting $\varpi = 0$ in the fundamental equation (3.42), one obtains

$$\ddot{Q}_n(t) + (1 - \frac{F_4^*}{\theta^*})\omega_n^2 Q_n(t) - \frac{1}{\mu\theta^*}(\mu R_0 T_1 \ddot{Q}_n(t) - 2BT_2 Q_n(t) - D_y T_3 Q_n(t) - D_x T_4 Q_n(t) - K_o T_5 Q_n(t) + G_0(T_6 + T_7) Q_n(t)) - \frac{1}{\mu\theta^*}\sum_{q=1}^{\infty} [\mu R_0 T_1 \ddot{Q}_q(t) - 2BT_2 Q_q(t) - D_y T_3 Q_q(t) - D_x F_4^* Q_q(t) + (\mu\omega_q^2 F_4^* - K_o T_5) Q_q(t) + G_0(T_6 + T_7) Q_q(t)] = \sum_{q=1}^{\infty}\sum_{r=1}^{N} \frac{Mg}{\mu\Theta^*}\Psi_m(ct)\Psi_m(s) \tag{3.47}$$

which can further be simplified as

$$\ddot{Q}_n(t) + \chi_n^2 Q_n(t) - \tau^*(\mu R_0 T_1 \ddot{Q}_n(t) - 2BT_2 Q_n(t) - D_y T_3 Q_n(t) - D_x T_4 Q_n(t) - K_o T_5 Q_n(t) + G_0(T_6 + T_7) Q_n(t)) - \tau^* \sum_{q=1}^{\infty} [\mu R_0 T_1 \ddot{Q}_q(t) - 2BT_2 Q_q(t) - D_y T_3 Q_q(t) - D_x T_4 Q_q(t) + (\mu\omega_q^2 F_4^* - K_o T_5) Q_q(t) + G_0(T_6 + T_7) Q_q(t)] = \sum_{q=1}^{\infty}\sum_{r=1}^{N} Mg\tau^*\Psi_m(ct)\Psi_m(s) \tag{3.48}$$

On expanding, re-arranging and simplifying equation (4.48), one obtains

$$\ddot{Q}_n(t) + \frac{(\chi_n^2 - \tau^* J_6^*)}{[1 - \tau^*\mu R_0 T_1]} Q_n(t) + \frac{\tau}{[1 - \tau^*\mu R_0 T_1]}\sum_{q=1, q\neq n}^{\infty} (\mu R_0 T_1 \ddot{Q}_q(t) - 2BT_2 Q_q(t) - D_y T_3 Q_q(t) - D_x T_4 Q_q(t) + (\mu\omega_q^2 F_4^* - K_o T_5) Q_q(t) + G_0(T_6 + T_7) Q_q(t)) = \frac{\tau^* Mg}{[1 - \tau^*\mu R_0 F_1^*]}\Psi_m(ct)\Psi_m(s) \tag{3.49}$$

where

$$\tau^* = \frac{1}{\mu\theta^*}, \quad \chi_n^2 = (1 - \frac{F_4^*}{\theta^*})\omega_n^2, \quad J_6^* = -2BT_2 - D_yT_3 - D_xT_4 - K_0T_5 + G_0(T_6 + T_7) \tag{3.50}$$

For any arbitrary ratio Υ, defined as $\tau^* = \frac{\Upsilon}{1+\Upsilon}$, one obtains

$$\Upsilon = \frac{\tau^*}{1-\tau^*} = \tau^* + o(\tau^{*2}) + \ldots \tag{3.51}$$

For only $o(\tau^*)$, one obtains

$$\tau^* = \Upsilon \tag{3.52}$$

Applying binomial expansion,

$$\frac{1}{1-\Upsilon\mu R_0F_1^*} = 1 + \Upsilon\mu R_0F_1^* + o(\Upsilon^2) + \ldots \tag{3.53}$$

On putting equation (4.53) into equation (4.49), one obtains

$$\ddot{Q}_n(t) + (\chi_n^2 - \Upsilon J_6^*)(1 + \Upsilon\mu R_0T_1 + o(\Upsilon^2) + \ldots)Q_n(t) + \Upsilon(1 + \tau^*\mu R_0T_1 + o(\Upsilon^2) + \ldots) \sum_{q=1,q\neq n}^{\infty} (\mu R_0F_1^*\ddot{Z}_q(t) - 2BF_2^*Q_q(t) - D_yF_3^*Q_q(t) - D_xF_4^*Q_q(t) + (\mu\omega_q^2 - K_0)F_5^*Q_q(t) + G_0(F_6^* + F_7^*)Q_q(t)) = (1 + \Upsilon\mu R_0T_1 + o(\Upsilon^2) + \ldots)Mg\Psi_m(ct)\Psi_m(s) \tag{3.54}$$

Retaining only $o(\Upsilon)$, equation (4.54) becomes

$$\ddot{Q}_n(t) + (\chi_n^2(1 + \Upsilon\mu R_0T_1) - \Upsilon J_6^*)Q_n(t) + \Upsilon \sum_{q=1,q\neq n}^{\infty} (\mu R_0F_1^*\ddot{Q}_q(t) - 2BF_2^*Q_q(t) - D_yF_3^* Q_q(t) - D_xF_4^*Q_q(t) + (\mu\omega_q^2 - K_0)F_5^*Q_q(t) + G_0(F_6^* + F_7^*)Q_q(t)) = \Upsilon Mg\Psi_m(ct)\Psi_m(s) \tag{3.55}$$

which is simplified further as

$$\ddot{Q}_n(t) + J_7^*Q_n(t) + \Upsilon \sum_{q=1,q\neq n}^{\infty} (\mu R_0F_1^*\ddot{Q}_q(t) - 2BF_2^*Q_q(t) - D_yF_3^*Q_q(t) - D_xF_4^*Q_q(t) + (\mu\omega_q^2 - K_0)F_5^*Q_q(t) + G_0(F_6^* + F_7^*)Q_q(t)) = \Upsilon Mg\Psi_m(ct)\Psi_m(s) \tag{3.56}$$

where

$$J_7^* = \chi_n^2(1 + \Upsilon\mu R_0T_1) - \Upsilon J_6^* \tag{3.57}$$

Using Struble's technique, the solution to the homogeneous part of part of equation (3.57) is assumed to take the form

$$Q_n(t) = \varphi(n,t)\cos(\chi_n t - \rho(n,t)) + \ldots \tag{3.58}$$

where

$$\varphi(n,t) = \varepsilon_n \tag{3.59}$$

and

$$\phi(n,t) = (\frac{\chi_n^2 - J_7^*}{2\chi_n})t + \iota_n \tag{3.60}$$

On putting equations (3.59) and (3.60) into equation (3.58), one obtains

$$Q_n(t) = \varepsilon_n\cos(\chi_n t - (\frac{\chi_n^2 - J_7^*\varphi(n,t)}{2\chi_n})t - \iota_n) \tag{3.61}$$

On further simplification, one obtains

$$Q_n(t) = \varepsilon_n\cos(\upsilon_n t - \iota_n) \tag{3.62}$$

where

$$\upsilon_n = \chi_n - (\frac{\chi_n^2 - J_7^*\varphi(n,t)}{2\chi_n}) \tag{3.63}$$

is the modified frequency for moving force problem for orthotropic rectangular plate resting on variable elastic bi-parametric foundation.

Using equation (3.63), the homogeneous part of equation (3.56)

$$\ddot{Q}_n(t) + \upsilon_n^2Q_n(t) = 0 \tag{3.64}$$

Hence, the entire equation (3.56) gives

$$\ddot{Q}_n(t) + \upsilon_n^2Q_n(t) = \Upsilon Mg\Psi_m(ct)\Psi_m(s) \tag{3.65}$$

Re-writing equation (3.65), one obtains

$$\ddot{Q}_n(t) + \upsilon_n^2 Q_n(t) = \Upsilon M g \Psi_m(s)[\sin\chi_m(t) + A_m\cos\chi_m t + B_m\sinh\chi_m t + C_m\cosh\chi_m t] \quad (3.66)$$

To obtain the solution to equation (3.66), one makes use of Laplace transformation techniques to obtain

$$Q_n(t) = \frac{Mg\Upsilon\Phi_m(s)}{\upsilon_n(\chi_m^4-\upsilon_n^4)}[(\chi_m^2+\upsilon_n^2)(\chi_m\sin\upsilon_n t - \upsilon_n\sin\chi_m t) - A_m\upsilon_n(\chi_m^2+\upsilon_n^2)(\cos\chi_m t - \cos\upsilon_n t) - B_m(\chi_m^2-\upsilon_n^2)(\alpha_m\sin\upsilon_n t - \upsilon_n\sinh\chi_m t) + C_m\upsilon_n(\chi_m^2-\upsilon_n^2)(\cosh\chi_m t - \cos\upsilon_n t)] \quad (3.67)$$

which on inversion yields

$$W(x,y,t) = \sum_{jm=1}^{\infty}\sum_{hm=1}^{\infty}\frac{Mg\Upsilon\Phi_m(s)}{\upsilon_n(\chi_m^4-\upsilon_n^4)}[(\chi_m^2+\upsilon_n^2)(\chi_m\sin\upsilon_n t - \upsilon_n\sin\chi_m t) - A_m\upsilon_n(\chi_m^2+\upsilon_n^2)(\cos\chi_m t - \cos\upsilon_n t) - B_m(\chi_m^2-\upsilon_n^2)(\chi_m\sin\upsilon_n t - \upsilon_n\sinh\chi_m t) + C_m\upsilon_n(\chi_m^2-\upsilon_n^2)(\cosh\chi_m t - \cos\upsilon_n t)](\sin\frac{\phi_{jm}}{L_x}x + A_{jm}\cos\frac{\phi_{jm}}{L_x}x + B_{jm}\sinh\frac{\phi_{jm}}{L_x}x + C_{jm}\cosh\frac{\phi_{jm}}{L_x}x)(\sin\frac{\phi_{hm}}{L_y}y + A_{hm}\cos\frac{\phi_{hm}}{L_y}y + B_{hm}\sinh\frac{\phi_{hm}}{L_y}y + C_{hm}\cosh\frac{\phi_{hm}}{L_y}y) \quad (3.68)$$

which is the transverse displacement response to a moving force of orthotropic rectangular plate resting on variable elastic bi-parametric foundation.

3.2 Orthotropic Rectangular Plate Traversed by a Moving Mass

In moving mass problem, the moving load is assumed rigid, and the weight and as well as inertia forces are transferred to the moving load. That is the inertia effect is not negligible. Thus $\varpi \neq 0$ and so it is required to solve the entire equation (3.42).To solve the equation,one employs analytical approximate method. This method is known as an approximate analytical method of Struble. The homogeneous part of equation (3.42) shall be replaced by a free system operator defined by the modified frequency υ_n. Thus, the entire equation becomes

$$\ddot{Q}_n(t) + \upsilon_n^2 Q_n(t) + \alpha\rho^*\sum_{q=1}^{\infty}[(F_8^* + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_1^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty}E_2^*\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty}E_3^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_4^*\frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}E_5^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty}E_6^*\frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty}E_7^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_8^*\frac{\sin(2k+1)\pi s}{2k+1}))\ddot{Q}_q(t) + 2c(F_9^* + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_9^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty}E_{10}^*\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty}E_{11}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{j=1}^{\infty}E_{12}^*\frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}E_{13}^*\frac{\cos(2j+1)\pi ct}{2j+1}$$

$$- \sum_{j=1}^{\infty}E_{14}^*\frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty}E_{15}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_{16}^*\frac{\sin(2k+1)\pi s}{2k+1}))\dot{Q}_q(t) + c^2(F_{10}^* + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_{17}^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty}E_{18}^*\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty}E_{19}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_{20}^*\frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}E_{21}^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty}E_{22}^*\frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty}E_{23}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}E_{24}^*\frac{\sin(2k+1)\pi s}{2k+1}))Q_q(t)] = \sum_{q=1}^{\infty}\frac{g\alpha}{\theta^*}\Phi_m(ct)\Phi_m(s) \quad (3.69)$$

where $\rho^* = \frac{\rho}{\theta^*}$

On expanding,simplifying and rearranging equation (4.88), one obtains

$$\ddot{Q}_n(t) + 2c\alpha\rho^*(F_9^* + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_9^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty}E_{10}^*\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty}E_{11}^*\frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty}E_{12}^*\frac{\sin(2k+1)\pi ct}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}E_{13}^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty}E_{14}^*\frac{\sin(2j+1)\pi ct}{2j+1}) + (\sum_{k=1}^{\infty}E_{15}^*\frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty}E_{16}^*\frac{\sin(2k+1)\pi ct}{2k+1}))\dot{Q}_n(t) + (\upsilon_n^2(1 - \alpha\rho^*(F_8^* + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_1^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty}E_2^*\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty}E_3^*\frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty}E_4^*\frac{\sin(2k+1)\pi ct}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}E_5^*\frac{\cos(2j+1)\pi ct}{2j+1} -$$

$$\sum_{j=1}^{\infty} E_6^* \frac{\sin(2j+1)\pi ct}{2j+1}) + (\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi ct}{2k+1})))) +$$
$$c^2\alpha\rho^*(F_{10}^* + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_{19}^*$$
$$\frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \frac{\sin(2k+1)\pi ct}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{22}^*$$
$$\frac{\sin(2j+1)\pi ct}{2j+1}) + (\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1}))Q_n(t)$$
$$+\alpha\rho^* \sum_{q=1,q\neq n}^{\infty} [(F_8^* + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_2^* \frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_3^*$$
$$\frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi ct}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_5^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_6^*$$
$$\frac{\sin(2j+1)\pi ct}{2j+1}) + (\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi ct}{2k+1}))\ddot{Q}_q(t) +$$
$$2c(F_9^* + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_{11}^* \frac{\cos(2k+1)\pi ct}{2k+1}$$
$$- \sum_{k=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi ct}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{13}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{14}^* \frac{\sin(2j+1)\pi ct}{2j+1})$$
$$+(\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi ct}{2k+1}))\dot{Q}_q(t) + c^2(F_{10}^* + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_{17}^*$$
$$\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{20}^*$$

$$\frac{\sin(2k+1)\pi ct}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}$$
$$(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1}))Q_q(t)] = \frac{g\alpha}{\theta^*}\Phi_m(ct)\Phi_m(s) \tag{3.70}$$

Applying modified asymptotic method of Struble, the solution to equation (3.70) takes the form

$$Q_n(t) = \psi(n,t)\cos(\upsilon_n t - G(n,t)) + \upsilon_n Q_1(t) + \ldots \tag{3.71}$$

where

$$\psi(n,t) = \beta e^{-\alpha\rho^* J_8 t} \tag{3.72}$$

and

$$F(n,t) = \frac{1}{2\upsilon_n}[\upsilon_n^2 \varpi\varphi^*(F_9^* + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi s}{2k+1}))$$
$$-c^2\varpi\varphi^*(F_{10}^* + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1}))]t + \tau_n \tag{3.73}$$

On putting equations (3.72) and (3.73) into equation (3.71), one obtains

$$Q_n(t) = Ze^{-\alpha\rho^* J_8 t}\cos[\upsilon_n t - \frac{1}{2\upsilon_n}(\upsilon_n^2 \alpha\rho^*(F_8^* + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^*$$
$$\frac{\sin(2k+1)\pi s}{2k+1})) - c^2\alpha\rho^*(F_{10}^* + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1})$$
$$))t - \tau_n] \tag{3.74}$$

On further simplifications, one obtains

$$Q_n(t) = Ze^{-\alpha\rho^* J_8 t}\cos[\beta_n t - \varepsilon_n] \tag{3.75}$$

where

$$\beta_n = \upsilon_n - \frac{1}{2\upsilon_n}(\upsilon_n^2 \alpha\rho^*(F_8^* + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi s}{2k+1}))$$
$$-c^2\alpha\rho^*(F_{10}^* + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1}))) \tag{3.76}$$

is the modified frequency representing the frequency of the free system.

Using the same argument used to solve equation (3.56), the homogeneous part of equation (3.70) becomes

$$\ddot{Q}_n(t) + \beta_n^2 Q_n(t) = 0 \tag{3.77}$$

Hence, the entire equation becomes

$$\ddot{Q}_n(t) + \beta_n^2 Q_n(t) = \frac{g\alpha}{\theta^*}\Phi_m(ct)\Phi_m(s) \tag{3.78}$$

Rewriting equation (3.78), one obtains

$$\ddot{Q}_n(t) + \beta_n^2 Q_n(t) = \frac{g\alpha}{\theta^*}\Phi_m(s)[\sin\chi_m t + A_m\cos\chi_m t + B_m\sinh\chi_m t + C_m\cosh\chi_m t] \tag{3.79}$$

Following the procedures applied to solve equation (3.66), one obtains

$$Q_n(t) = \frac{g\alpha\Phi_m(s)}{\theta^*\beta_n(\chi_m^4-\beta_n^4)}[(\chi_m^2+\beta_n^2)(\chi_m\sin\beta_n t - \beta_n\sin\chi_m t) - A_m\beta_n(\chi_m^2+\beta_n^2)(\cos\chi_m t - \cos\beta_n t) - B_m(\chi_m^2-\beta_n^2)(\chi_m\sin\beta_n t - \beta_n\sinh\chi_m t) + C_m\beta_n(\chi_m^2-\beta_n^2)(\cosh\chi_m t - \cos\beta_n t)] \tag{3.80}$$

which on inversion yields

$$W(x,y,t) = \sum_{jm=1}^{\infty}\sum_{hm=1}^{\infty}\frac{g\alpha\Phi_m(s)}{\theta^*\beta_n(\chi_m^4-\beta_n^4)}[(\chi_m^2+\beta_n^2)(\chi_m\sin\beta_n t - \beta_n\sin\chi_m t) - A_m\beta_n(\chi_m^2+\beta_n^2)(\cos\chi_m t - \cos\beta_n t) - B_m(\chi_m^2-\beta_n^2)(\chi_m\sin\beta_n t - \beta_n\sinh\chi_m t) + C_m\beta_n(\chi_m^2-\beta_n^2)(\cosh\chi_m t - \cos\beta_n t)](\sin\frac{\phi_{jm}}{L_x}x + A_{jm}\cos\frac{\phi_{jm}}{L_x}x + B_{jm}\sinh\frac{\phi_{jm}}{L_x}x + C_{jm}\cosh\frac{\phi_{jm}}{L_x}x)(\sin\frac{\phi_{hm}}{L_y}y + A_{hm}\cos\frac{\phi_{hm}}{L_y}y + B_{hm}\sinh\frac{\phi_{hm}}{L_y}y + C_{hm}\cosh\frac{\phi_{hm}}{L_y}y) \tag{3.81}$$

which is the transverse displacement response to a moving mass of an orthotropic rectangular plate resting on variable elastic bi-parametric foundation.

3.3 ILLUSTRATIVE EXAMPLES

3.3.1 Orthotropic Rectangular Plate Clamped at All Edges

For an orthotropic plate clamped at all its edges, the boundary conditions are given by

$$W(0,y,t) = 0, \quad W(L_x,y,t) = 0 \tag{3.82}$$

$$W(x,0,t) = 0, \quad W(x,L_y,t) = 0 \tag{3.83}$$

$$\frac{\partial W(0,y,t)}{\partial x} = 0, \quad \frac{\partial W(L_x,y,t)}{\partial x} = 0 \tag{3.84}$$

$$\frac{\partial W(x,0,t)}{\partial y} = 0, \quad \frac{\partial W(x,L_y,t)}{\partial y} = 0 \tag{3.85}$$

Thus,for the normal modes

$$\xi_{pm}(0) = 0, \quad \xi_{pm}(L_x) = 0 \tag{3.86}$$

$$\xi_{qm}(0) = 0, \quad \xi_{qm}(L_y) = 0 \tag{3.87}$$

$$\frac{\partial\xi_{pm}(0)}{\partial x} = 0, \quad \frac{\partial\xi_{pm}(L_x)}{\partial x} = 0 \tag{3.88}$$

$$\frac{\partial\xi_{qm}(0)}{\partial y} = 0, \quad \frac{\partial\xi_{qm}(L_y)}{\partial y} = 0 \tag{3.89}$$

For simplicity, our initial conditions are of the form

$$W(x,y,0) = 0 = \frac{\partial W(x,y,o)}{\partial t} \tag{3.90}$$

Using the boundary conditions in equations (3.86) to (3.89) and the initial conditions given by equation (3.90), it can be shown that

$$A_{pm} = \frac{\sinh\xi_{pm} - \sin\xi_{pm}}{\cos\xi_{pm} - \cosh\xi_{pm}} = \frac{\cos\xi_{pm} - \cosh\xi_{pm}}{\sin\xi_{pm} + \sinh\xi_{pm}} \tag{3.91}$$

$$A_{qm} = \frac{\sinh\xi_{qm} - \sin\xi_{qm}}{\cos\xi_{qm} - \cosh\xi_{qm}} = \frac{\cos\xi_{qm} - \cosh\xi_{qm}}{\sin\xi_{qm} + \sinh\xi_{qm}} \qquad (3.92)$$

In the same vein, we have

$$A_m = \frac{\sinh\xi_m - \sin\xi_m}{\cos\xi_m - \cosh\xi_m} = \frac{\cos\xi_m - \cosh\xi_m}{\sin\xi_m + \sinh\xi_m} \qquad (3.93)$$

$$B_{pm} = -1, \quad B_{qm} = -1, \quad \Rightarrow B_m = -1 \qquad (3.94)$$

$$C_{pm} = -A_{pm}, \quad C_{qm} = -A_{qm}, \quad \Rightarrow C_m = -A_m \qquad (3.95)$$

and from equation (3.93) , one obtains

$$\cos\xi_m \cosh\xi_m = 1 \qquad (3.96)$$

which is termed the frequency equation for the dynamical problem, such that

$$\xi_1 = 4.73004, \quad \xi_2 = 7.85320, \quad \xi_3 = 10.9951 \qquad (3.97)$$

On using equations (3.91), (3.92), (3.93),(3.94),(3.95) and (3.97) in equations (3.68) and (3.81), one obtains the displacement response to a moving force and a moving mass of clamped orthotropic rectangular plate resting on bi-parametric condition respectively.

3.3.2 Graphs of the Clamped-clamped End Conditions

Figures 1 and 2 display the effect of foundation modulus K_o on the deflection profile of clamped-clamped orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of foundation modulus K_o increases.

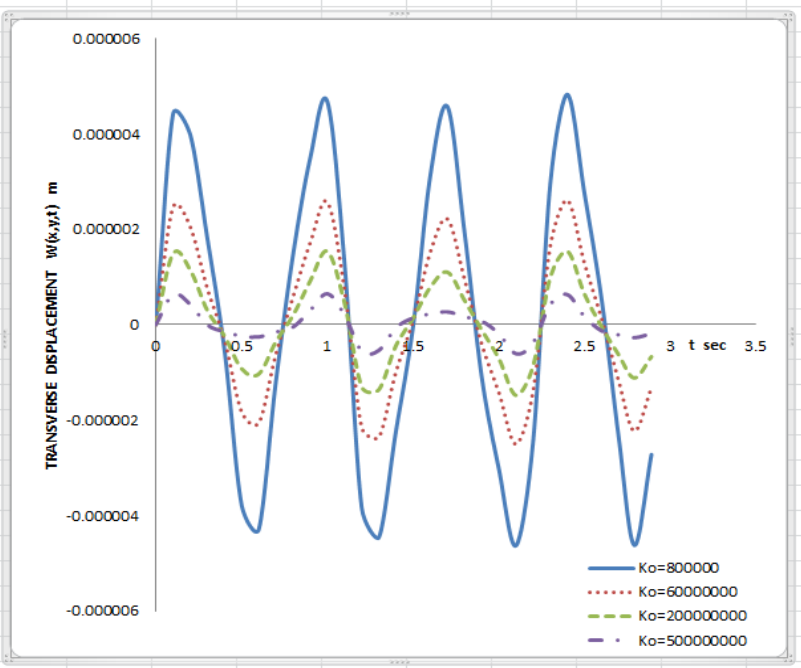

Fig.1: Displacement Profile of Clamped-clamped Orthotropic Rectangular Plate with Varying K_o and Traversed by Moving Force

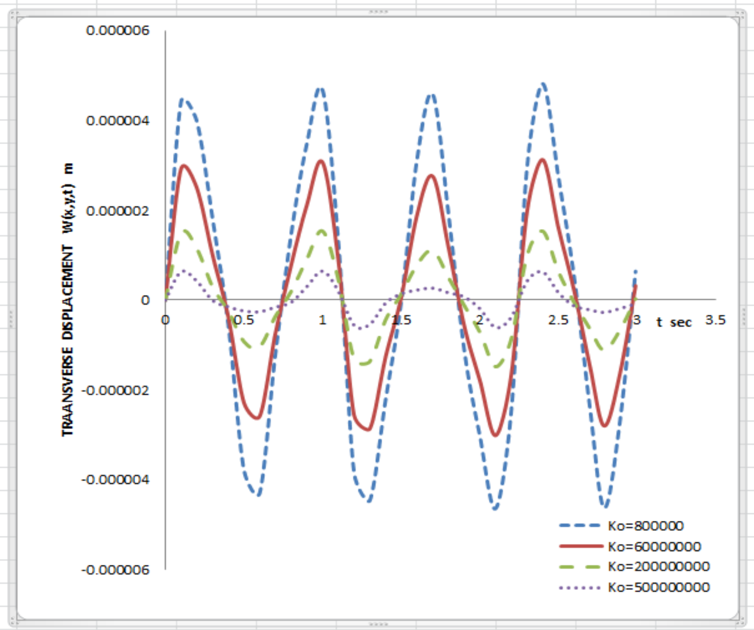

Fig.2: Displacement Profile of Clamped-clamped Orthotropic Rectangular Plate with Varying K_o and Traversed by Moving Mass

Figures 3 and 4 display the effect of shear modulus G_o on the deflection profile of clamped-clamped orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of shear modulus G_o increases.

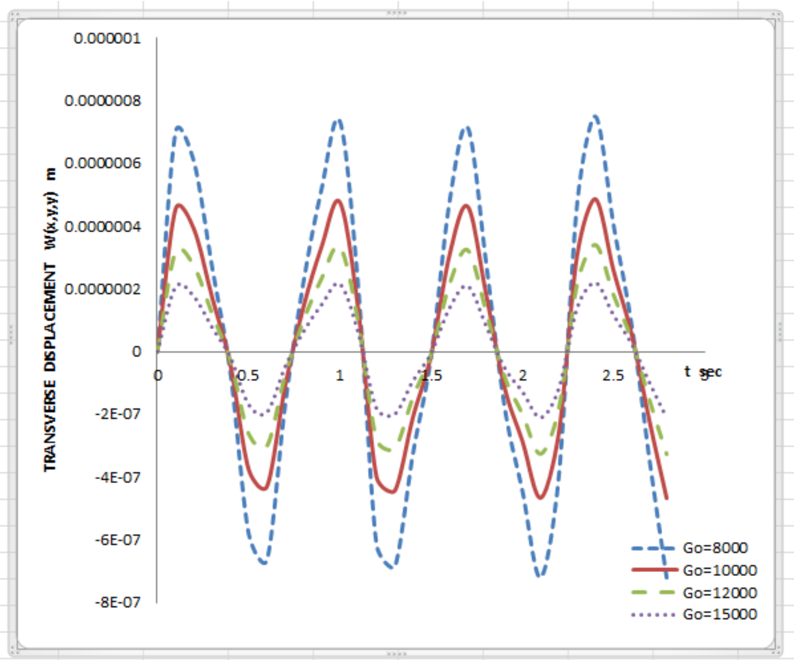

Fig.3: Displacement Profile of Clamped-clamped Orthotropic Rectangular Plate with Varying G_o and Traversed by Moving Force

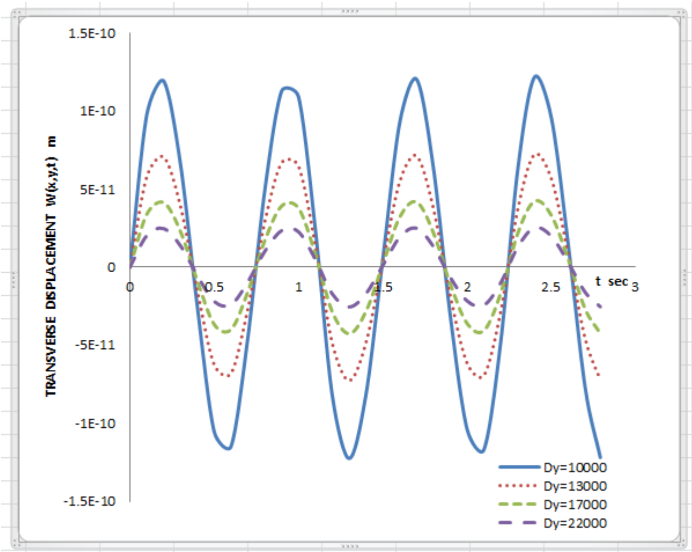

Fig.4: Displacement Profile of Clamped-clamped Orthotropic Rectangular Plate with Varying G_o and Traversed by Moving Mass

Figures 5 and 6 display the effect of flexural rigidity of the plate along x-axis D_x on the deflection profile of Clamped-clamped orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of flexural rigidity D_x increases.

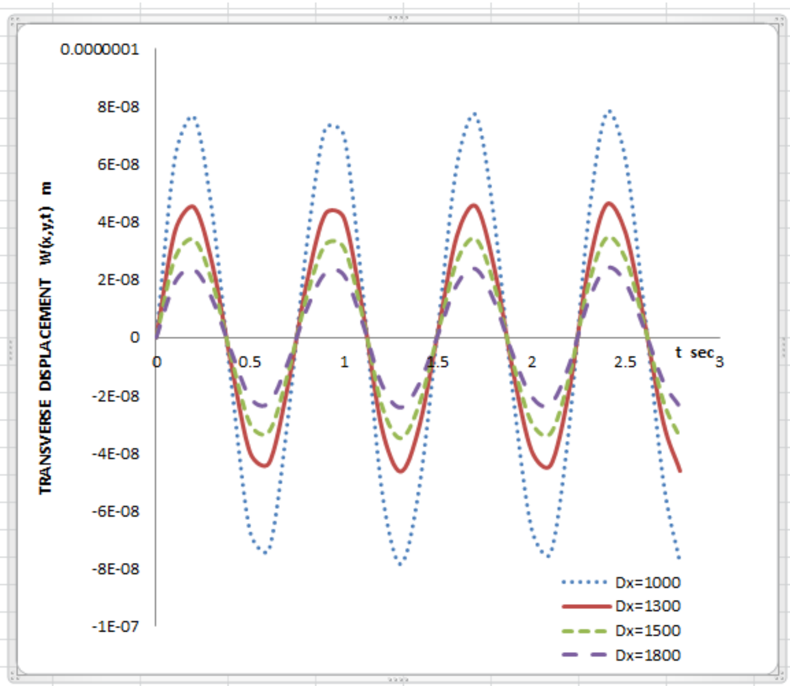

Fig.5: Displacement Profile of Clamped-clamped Orthotropic Rectangular Plate with Varying D_x and Traversed by Moving Force

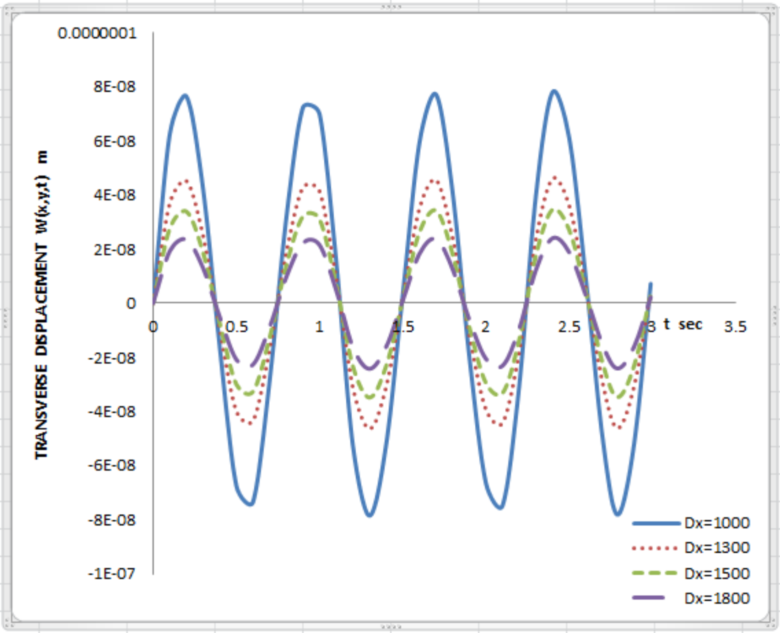

Fig.6: Displacement Profile of Clamped-clamped Orthotropic Rectangular Plate with Varying D_x *and Traversed by Moving Mass*

Figures 7 and 8 display the effect of flexural rigidity of the plate along y-axis D_y on the deflection profile of Clamped-clamped orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of flexural rigidity D_y increases.

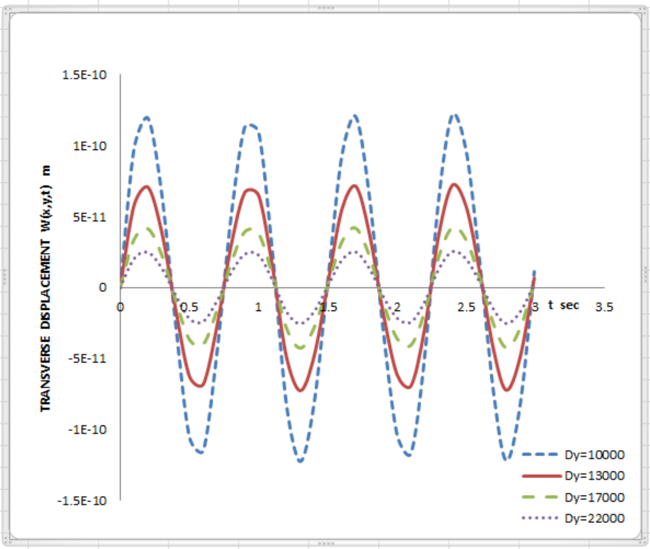

Fig.7: Displacement Profile of Clamped-clamped Orthotropic Rectangular Plate with Varying D_y *and Traversed by Moving Force*

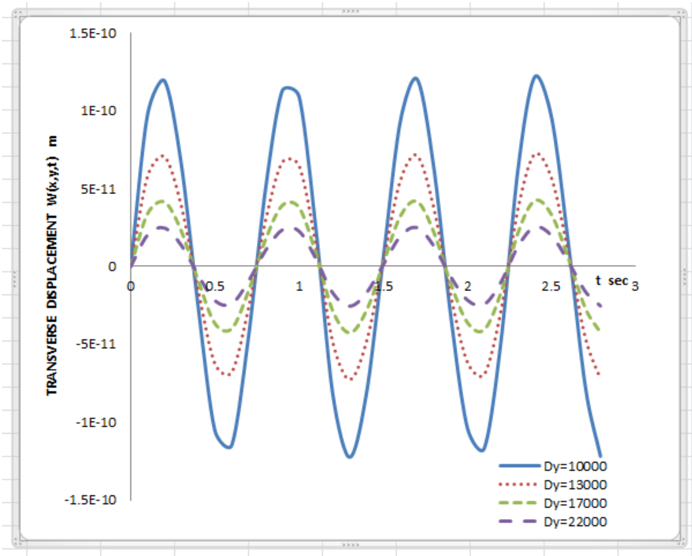

Fig.8: Displacement Profile of Clamped-clamped Orthotropic Rectangular Plate with Varying D_y and Traversed by Moving Mass

Figures 9 and 10 display the effect of rotatory inertia R_o on the deflection profile of Clamped-clamped orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of rotatory inertia R_o increases.

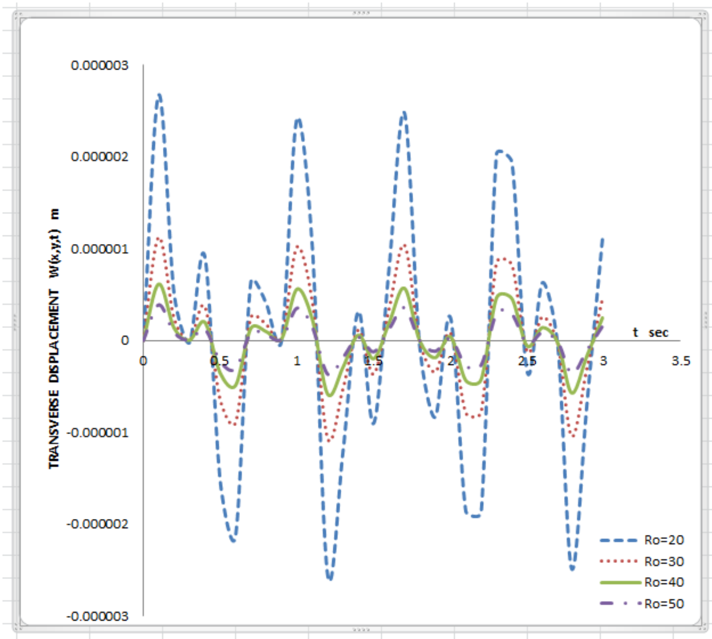

Fig.9: Displacement Profile of Clamped-clamped Orthotropic Rectangular Plate with Varying R_o and Traversed by Moving Force

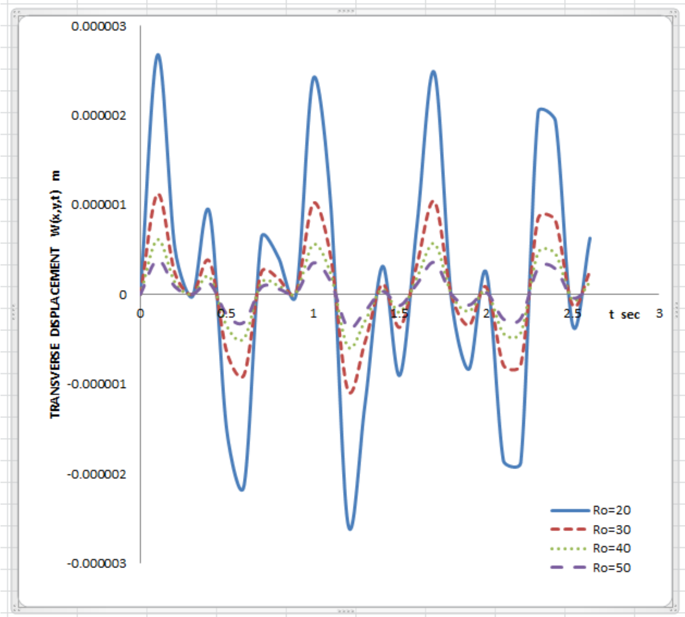

Fig.10: Displacement Profile of Clamped-clamped Orthotropic Rectangular Plate with Varying R_o *and Traversed by Moving Mass*

Figure 11 displays the comparison between moving force and moving mass for fixed values of R_o, G_o, K_o, D_x and D_y.

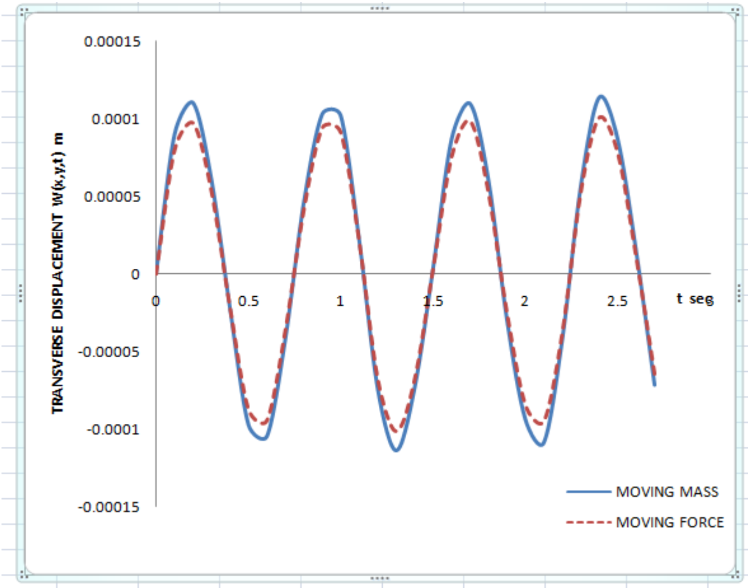

Fig.11: Displacement Profile of Comparison between Moving Force and Moving Mass

IV. CONCLUSION

In this research work, the problem of vibration of orthotropic rectangular plate under the action of moving masses and resting on a variable elastic Pasternak foundation with clamped end conditions has been studied. The closed form solutions of the fourth order partial differential equations with variable and singular coefficients governing the orthotropic rectangular plates is obtained for both cases of moving force and moving mass using a solution technique that is based on the separation of variables which was used to remove the singularity in the governing fourth order partial differential equation and thereby reducing it to a sequence of coupled second order differential equations. The modified asymptotic method of Struble and Laplace transformation techniques are then employed to obtain the analytical solution to the two-dimensional dynamical problem.

The solutions are then analyzed. The analyses show that, for

the same natural frequency and the critical speed, the moving mass problem is smaller than that of the moving force problem. Resonance is reached earlier in the moving mass system than in the moving force problem. That is to say the moving force solution is not an upper bound for the accurate solution of the moving mass problem.

The results in plotted curves show that as foundation modulus K_o and the shear modulus G_o increase, the amplitudes of plates decrease for both cases of moving force and moving mass problems. As the rotatory inertia correction factor R_o increases, the amplitudes of plates decrease for both cases of moving force and moving mass problems. As the flexural rigidities along both the x-axis D_x and y-axis D_y increase, the amplitudes of plates decrease for both cases of moving force and moving mass problems.

It can be shown further from the results that for fixed values of foundation modulus and shear modulus, rotatory inertia correction factor, flexural rigidities along both x-axis and y-axis,the amplitude for the moving mass problem is greater than that of the moving force problem which implies that resonance is reached earlier in moving mass problem than in moving force problem of simply supported orthotropic rectangular plates resting on bi-parametric foundation.

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